

On D-brane -Anti D-brane Effective actions and their all order Bulk Singularity Structures

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Abstract

All four point functions of brane anti brane system including their correct all order α' corrections have been addressed. All five point functions of one closed string Ramond-Ramond (RR), two real tachyons and either one gauge field or the scalar field in both symmetric and asymmetric pictures have also been explored. The entire analysis of $\langle C^{-2}A^0T^0T^0 \rangle$ is carried out. Not only does it fix the vertex operator of RR in asymmetric picture and in higher point functions of string theory amplitudes but also it confirms the fact that there is no issue of picture dependence of the mixed closed RR, gauge fields, tachyons and fermion fields in all symmetric or anti symmetric ones. We compute $\langle C^{-2}\phi^0T^0T^0 \rangle$ S-matrix in the presence of a transverse scalar field, two real tachyons and that reveals two different kinds of bulk singularity structures, involving an infinite number of u - channel gauge field and $(u + s' + t')$ -channel scalar bulk poles. In order to produce all those bulk singularity structures, we define various couplings at the level of the effective field theory that involve the mixing term of Chern-Simons coupling (with C-potential field) and a covariant derivative of the scalar field that comes from the pull-back of brane. Eventually we explore their all order α' corrections in the presence of brane anti brane system where various remarks will be also pointed out.

1 Introduction

By studying the unstable branes in detail , one may hope to have some sort of understanding about supersymmetry breaking as well as getting more knowledge about at least some of the string theory 's properties in the backgrounds which are specially time dependent [1, 2, 3].

Having dealt with unstable branes , one could also talk about Sakai- Sugimoto model [4], the low energy hadron physics in some of the holographic QCD models as well as studying properly the so called spontaneous symmetry breaking in some of the holographic QCD models [5]. Indeed we try to define flavour branes by embedding various parallel branes and consider all anti branes inside a background which must be dual to some theory such as colour confined theorem. In fact one might consider brane anti brane like system as a probe if and only if $N_f \ll N_c$. Strictly speaking, one reveals the fact that the presence of tachyons in the spectrum is the only source of instability in all these formalisms, therefore it absolutely makes sense to actually work out within details with some field theories that do include these unstable modes.

It was A.Sen who realized an effective actions [6, 7, 8] that involves tachyons, can describe various issues such as the decays [9] of the odd parity branes or the so called non-BPS branes in superstring theory. It is also shown in [3] that the Effective Field Theory (EFT) for all non-BPS branes includes just massless states as well as tachyons as we have dealt with non-BPS effective actions in [10] where various remarks on tachyon condensation for brane anti brane system have also been presented in [11].

Sen has also clarified for brane anti brane system in [12] that once the separation of branes becomes smaller than string length scale then two real tachyon modes show up in the spectrum of brane anti brane , hence it makes sense to substitute them in an effective action of brane anti brane system as they are going to play for us the most fundamental role in ascribing the dynamics of the configuration.

Recently in [13] various remarks about the dynamic of brane anti brane system have been made, namely it is discussed that if one employs the brane actions in the context of EFT , then one is able to even properly deal with loop divergences.

Let us address various motivations or applications for brane anti brane systems .

Brane production [14], dealing with stable nonBPS D-branes in all type I,II string theory as well as having inflation in string theory in the language of brane anti brane or KKLT

are suggested [15].

Note that within just S-Matrix computations one assured that not only various new couplings can be verified but also all the coefficients of the string theory couplings are exact and appear without any ambiguity.¹ Just as an extra comment, one could consider the thermodynamical aspects of brane anti brane action, because it is known that at finite temperature this D-brane anti D-brane could be stabilised and is kind of related to black holes where to our knowledge some of the applications to AdS/CFT or to M-theory [20] were appeared in the literature.

It is worth pointing out the fact that brane anti brane has been playing a very strong role for the stability of KKLT as well as in large volume scenario [21].

In [22] Polchinski explained the deep and close relationship between the D-branes and the closed string RR field where we just demonstrate the very main paper on the content of bound states of the branes [23]. One reason in favour of scattering amplitude calculations is indeed its strong potential in getting exact and correct α' corrections. Although there is no duality transformation for non-BPS branes, it is worth to stress the following remark. In fact it was shown that even for various BPS S-matrices all order α' corrections of DBI action with exact coefficients can not be fixed by just applying T-duality transformation. Indeed it is clarified that just with direct S-matrix calculations, one can hope to precisely gain all order α' corrections of DBI action with exact coefficients for instance one might look at [17] .

To be able to read off all the ingredients of either Wess-Zumino (WZ) or DBI effective actions , we suggest [24, 25].

The paper is based on the following contents. In section two, we do evaluate the amplitude of two real tachyons of brane anti brane and a C - field (the so called RR potential term in asymmetric picture of the closed string RR), where this S-matrix in symmetric picture, at leading order was computed in [26]. We expand it out and explore not only its low energy limit by re generating a gauge field pole but also find out its correct and exact all infinite α' higher derivative corrections of two tachyons and a C-field. Basically we generate all order α' contact terms of $C_{p-1} \wedge dT \wedge dT^*$. In section three, we compute $\langle C^{-2} A^0 T^0 T^0 \rangle$

¹ Several remarks on higher point functions of the string amplitudes [16] as well as their corrections in [17, 18] are made , we indeed derived all order α' corrections to BPS branes , where one might be interested in looking at the eventual conjecture [19] on α' corrections that has worked out surprisingly for both non-BPS and supersymmetric cases.

amplitude in detail and talk about its unique expansion, we then start comparing it with its symmetric result. The outcome for both symmetric and asymmetric picture is the same and for the first time we understand the fact that even in higher point functions of string theory there is no picture dependence for the mixed RR and world volume strings, such as gauge fields, tachyons and fermions but not scalar fields.

This evidently confirms that the vertex operator of RR in asymmetric picture is exact and complete and no extra potential terms needed to be added to RR potential vertex operator in asymmetric picture. It also confirms that for mixed world volume S-matrices of closed string RR- gauge field and tachyon, even in five and higher point functions there is no picture dependence at all. However, the story gets complicated for the higher point functions of the mixed closed string RR and scalar fields as we point out in detail in section four. In section five we point out $\langle C^{-2}\phi^0 T^0 T^0 \rangle$ as well as its all two different symmetric results. Two different kinds of singularity structures that carry the scalar product of momentum of C-field in the bulk and scalar polarization ($p^i \xi_{1i}$) will be obtained. These terms can not be derived by momentum conservation along the world volume of brane and we called them bulk singularities. In order to produce those bulk singularities in an effective field theory we propose a new sort of coupling as follows

$$\frac{1}{(p-1)!} \mu_p (2\pi\alpha')^2 \int_{\Sigma_{(p+1)}} \text{Tr} \left(C_{a_0 \dots a_{p-3}}^i F_{a_{p-2} a_{p-1}} D_{a_p} \phi^i \right) \quad (1)$$

where in above the scalar field comes from pull-back of brane and the Chern-Simons coupling has also been taken into account. Setting (1) and discovering its all order α' corrections, we are able to precisely produce an infinite number of u channel bulk singularities of this S-matrix. Eventually we introduce a new coupling as follows

$$(2\pi\alpha') \mu_p \frac{1}{(p+1)!} \int_{\Sigma_{p+1}} \epsilon^{a_0 \dots a_p} C_{a_0 \dots a_{p-1}}^i D_{a_p} \phi^i \quad (2)$$

where in the above coupling, the scalar field has been taken from pull back of brane. We then derive its vertex operator in an EFT and by making use of all order α' corrections of two tachyon two scalar couplings in the world volume of brane anti brane [27] we exactly reconstruct all infinite $(u + s' + t')$ - channel bulk poles of this string amplitude in the background of brane anti brane in an EFT as well. It is worth pointing out that, within direct scattering computations in [27], we have already found several new couplings, such as $D\phi^{1i} \cdot D\phi_{2i}$ in brane anti brane system that shall be used in this paper as well. Let us turn to actual details of the paper.

2 The $C^{-2} - T^0 - T^0$ S-matrix

In this section first we would like to just mention very briefly the effective actions of brane anti brane system and then start analyzing the higher point functions in that background.

All the effective actions of a $D_p\bar{D}_p$ -brane in both Type IIA(B) theory can be achieved by embedding tachyons in both DBI and Wess-Zumino (WZ) actions. One can consider just two non-BPS D-branes in Type IIB(A) and project them out, however, for the purpose of this paper we limit ourselves to two tachyons, either a gauge field or an scalar field and an RR where RR part comes from Chern-Simons action and the other fields do appear in DBI as follows [28]

$$S_{DBI} = - \int d^{p+1} \sigma \text{Tr} \left(V(\mathcal{T}) \sqrt{-\det(\eta_{ab} + 2\pi\alpha' F_{ab} + 2\pi\alpha' D_a \mathcal{T} D_b \mathcal{T})} \right), \quad (3)$$

Note that the trace in (3) must be symmetric for all the matrices such as $F_{ab}, D_a \mathcal{T}, \mathcal{T}$ inside the potential that have shown up in the action. ²

Suppose we use the ordinary trace, then (3) gets replaced by A. Sen 's action [29], if we try to make the kinetic term symmetrized and evaluate the trace, however, there is an important fact which one must be aware of as follows.

Indeed it is shown by direct analysis of scattering amplitude in [27, 28] that Sen's effective action is not consistent with string theory amplitudes. Tachyon 's consistent potential in the context of scattering amplitude is

$$V(|T|) = 1 + \pi\alpha' m^2 |T|^2 + \frac{1}{2}(\pi\alpha' m^2 |T|^2)^2 + \dots$$

with $m^2 = -1/(2\alpha')$ is tachyons's mass square and T_p is the tension of a p-brane. Notice that the expansion has given us a very consistent result for the potential of $V(|T|) = e^{\pi\alpha' m^2 |T|^2}$ that has been imposed by boundary string field theory (BSFT) as well [30].

Note that in an important paper [31] a sigma model approach to string theory effective actions with tachyons, has also been released. It is discussed in detail in [28] that only the above effective action which is based on the direct string theory S-matrix calculations

² The definitions for all the matrices are

$$F_{ab} = \begin{pmatrix} F_{ab}^{(1)} & 0 \\ 0 & F_{ab}^{(2)} \end{pmatrix}, D_a \mathcal{T} = \begin{pmatrix} 0 & D_a T \\ (D_a T)^* & 0 \end{pmatrix}, \mathcal{T} = \begin{pmatrix} 0 & T \\ T^* & 0 \end{pmatrix} \quad (4)$$

with $F_{ab}^{(i)} = \partial_a A_b^{(i)} - \partial_b A_a^{(i)}$ and $D_a T = \partial_a T - i(A_a^{(1)} - A_a^{(2)})T$.

can produce all the infinite poles and contact interactions where the mixing terms such as $F^{(1)} \cdot F^{(2)}$ and $D\phi^{(1)} \cdot D\phi^{(2)}$ play the crucial roles in getting consistent results between field theory and string amplitudes so that the Lagrangian that contributes to an S-matrix of a gauge field and two tachyons (in the presence of RR) does involve the following interactions [28]:

$$\begin{aligned} \mathcal{L}_{DBI} = & -T_p(2\pi\alpha') \left(m^2|T|^2 + DT \cdot (DT)^* - \frac{\pi\alpha'}{2} (F^{(1)} \cdot F^{(1)} + F^{(2)} \cdot F^{(2)}) \right) + T_p(\pi\alpha')^3 \\ & \times \left(\frac{2}{3} DT \cdot (DT)^* (F^{(1)} \cdot F^{(1)} + F^{(1)} \cdot F^{(2)} + F^{(2)} \cdot F^{(2)}) \right. \\ & + \frac{2m^2}{3} |\tau|^2 (F^{(1)} \cdot F^{(1)} + F^{(1)} \cdot F^{(2)} + F^{(2)} \cdot F^{(2)}) \\ & \left. - \frac{4}{3} ((D^\mu T)^* D_\beta T + D^\mu T (D_\beta T)^*) (F^{(1)\mu\alpha} F_{\alpha\beta}^{(1)} + F^{(1)\mu\alpha} F_{\alpha\beta}^{(2)} + F^{(2)\mu\alpha} F_{\alpha\beta}^{(2)}) \right) \end{aligned} \quad (5)$$

On the other hand the WZ action of brane anti brane system that defines for us the allowed couplings of RR field to gauge field is derived in [32]³

$$S = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \left(e^{i2\pi\alpha' F^{(1)}} - e^{i2\pi\alpha' F^{(2)}} \right), \quad (6)$$

In [33] it is shown how to impose the tachyons inside the actions also one may use the so called super-connection of non-commutative geometry [34, 35, 36] as follows

$$S_{WZ} = \mu_p \int_{\Sigma_{(p+1)}} C \wedge \text{STr} e^{i2\pi\alpha' \mathcal{F}} \quad (7)$$

⁴ where it is shown in [28] how to get to the curvature as below

$$i\mathcal{F} = \begin{pmatrix} iF^{(1)} - \beta^2|T|^2 & \beta(DT)^* \\ \beta DT & iF^{(2)} - \beta^2|T|^2 \end{pmatrix},$$

with $F^{(i)} = \frac{1}{2} F_{ab}^{(i)} dx^a \wedge dx^b$ and $DT = [\partial_a T - i(A_a^{(1)} - A_a^{(2)})T] dx^a$. Essentially one can extract the terms inside the WZ action (7) to reconstruct the following terms that are needed later

³ Note that C is a sum on RR potentials $C = \sum_n (-i)^{\frac{p-m+1}{2}} C_m$.

⁴ with some definitions for the curvature

$$\mathcal{F} = d\mathcal{A} - i\mathcal{A} \wedge \mathcal{A}$$

and the super-connection as

$$i\mathcal{A} = \begin{pmatrix} iA^{(1)} & \beta T^* \\ \beta T & iA^{(2)} \end{pmatrix},$$

on,

$$C \wedge \text{STr } i\mathcal{F} = C_{p-1} \wedge (F^{(1)} - F^{(2)}) \quad (8)$$

$$\begin{aligned} C \wedge \text{STr } i\mathcal{F} \wedge i\mathcal{F} &= C_{p-3} \wedge \left\{ F^{(1)} \wedge F^{(1)} - F^{(2)} \wedge F^{(2)} \right\} \\ &\quad + C_{p-1} \wedge \left\{ -2\beta^2 |T|^2 (F^{(1)} - F^{(2)}) + 2i\beta^2 DT \wedge (DT)^* \right\} \end{aligned} \quad (9)$$

Let us turn to the calculations.

First of all note that the space-time two point function or world sheet three point function of one closed string RR and a real tachyon of non-BPS branes in both asymmetric and symmetric pictures in type IIA and IIB has already been calculated in [37] as follows:

$$\begin{aligned} \mathcal{A}^{C^{-1}T^{-1}} &\sim -2i\text{Tr} (P_- \mathbb{H}_{(n)} M_p) \\ \mathcal{A}^{C^{-2}T^0} &\sim 2^{1/2} (2ik_{1a}) \text{Tr} (P_- \mathcal{C}_{(n-1)} M_p \gamma^a) \end{aligned}$$

Applying the momentum conservation on the world volume , $(k_1 + p)^a = 0$, making use of $p^a \mathcal{C}_{n-1} = \mathbb{H}_n$ we are able to produce the same result in both pictures. If we would actually use

$$2i\pi\alpha'\beta'\mu'_p \int C_p \wedge DT$$

coupling in field theory, then we would be able to precisely produce the whole S-matrix of string amplitude (in this case is just pure contact interaction) in effective theory as well. Let us turn to world sheet four point function.

Here we do want to make use of CFT methods [38] to actually gain the four point function of two real tachyons and one asymmetric closed string RR which makes sense in the presence of D-brane anti D-brane of type II systems. The structure of C^{-2} has been first pointed out in [39] and then later was accommodated by [40] so that

$$\mathcal{A}^{C^{-2}T^0T^0} \sim \int dx dy d^2 z \langle V_T^{(0)}(y) V_T^{(0)}(x) V_{RR}^{(-2)}(z, \bar{z}) \rangle, \quad (10)$$

where all the vertices including their Chan-Paton factors in brane anti brane system can be achieved as

$$\begin{aligned} V_T^{(0)}(y) &= \alpha' i k_1 \cdot \psi(y) e^{\alpha' i k_1 \cdot X(y)} \lambda \otimes \sigma_1 \\ V_{RR}^{(-2)}(z, \bar{z}) &= (P_- \mathcal{C}_{(n-1)} M_p)^{\alpha\beta} e^{-3\phi(z)/2} S_\alpha(z) e^{i\frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i\frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \otimes I \end{aligned} \quad (11)$$

where at disk level amplitude both tachyons must be replaced on the boundary while RR is located at the middle of the defined disk.

On-shell conditions as well as the other definitions are

$$\begin{aligned} p^2 &= 0, \quad k_1^2 = k_2^2 = 1/4 \\ P_- &= \frac{1}{2}(1 - \gamma^{11}), \quad \mathbb{H}_{(n)} = \frac{a_n}{n!} H_{\mu_1 \dots \mu_n} \gamma^{\mu_1} \dots \gamma^{\mu_n}, \\ (P_- \mathbb{H}_{(n)})^{\alpha\beta} &= C^{\alpha\delta} (P_- \mathbb{H}_{(n)})_{\delta}{}^{\beta}. \end{aligned} \quad (12)$$

where $n = 2, 4, a_n = i$ ($n = 1, 3, 5, a_n = 1$) holds for type IIA (type IIB).

Let us just deal with the holomorphic counterparts of world-sheet fields, to do so we apply the so called doubling trick, which means that the following change of variables has been taken into account

$$\tilde{X}^\mu(\bar{z}) \rightarrow D_\nu^\mu X^\nu(\bar{z}), \quad \tilde{\psi}^\mu(\bar{z}) \rightarrow D_\nu^\mu \psi^\nu(\bar{z}), \quad \tilde{\phi}(\bar{z}) \rightarrow \phi(\bar{z}), \quad \text{and} \quad \tilde{S}_\alpha(\bar{z}) \rightarrow M_\alpha{}^\beta S_\beta(\bar{z}),$$

with the aforementioned matrices as below

$$D = \begin{pmatrix} -1_{9-p} & 0 \\ 0 & 1_{p+1} \end{pmatrix}, \quad \text{and} \quad M_p = \begin{cases} \frac{\pm i}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \epsilon_{i_1 \dots i_{p+1}} & \text{for } p \text{ even} \\ \frac{\pm 1}{(p+1)!} \gamma^{i_1} \gamma^{i_2} \dots \gamma^{i_{p+1}} \gamma_{11} \epsilon_{i_1 \dots i_{p+1}} & \text{for } p \text{ odd} \end{cases}$$

Having set them, we are admitted from now on to indeed just work out with holomorphic parts of the propagators for all world sheet fields of X^μ, ψ^μ, ϕ as below

$$\begin{aligned} \langle X^\mu(z) X^\nu(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} \log(z - w), \\ \langle \psi^\mu(z) \psi^\nu(w) \rangle &= -\frac{\alpha'}{2} \eta^{\mu\nu} (z - w)^{-1}, \\ \langle \phi(z) \phi(w) \rangle &= -\log(z - w). \end{aligned} \quad (13)$$

The CP factor for the above S-matrix is now $2\text{Tr}(\lambda_1 \lambda_2)$. Having replaced the vertices inside the S-matrix, the amplitude can be written down as

$$\begin{aligned} &-2\text{Tr}(\lambda_1 \lambda_2) \alpha'^2 k_{1a} k_{2b} \int dx_1 dx_2 dx_4 dx_5 (x_{45})^{-3/4} I_1(P_- \mathcal{C}_{(n-1)} M_p)^{\alpha\beta} \\ &\times <: S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_1) : \psi^b(x_2) : >, \end{aligned}$$

where $x_4 = z = x + iy, x_5 = \bar{z} = x - iy$ and $x_{ij} = x_i - x_j$

$$I_1 = |x_{12}|^{\alpha'^2 k_1 \cdot k_2} |x_{14} x_{15}|^{\frac{\alpha'^2}{2} k_1 \cdot p} |x_{24} x_{25}|^{\frac{\alpha'^2}{2} k_2 \cdot p} |x_{45}|^{\frac{\alpha'^2}{4} p \cdot D \cdot p}, \quad (14)$$

One has to use the modified Wick-like rule [41] to be able to derive the following correlation function

$$\begin{aligned} \langle : S_\alpha(x_4) : S_\beta(x_5) : \psi^a(x_1) : \psi^b(x_2) : \rangle &= \left((\Gamma^{ba} C^{-1})_{\alpha\beta} - 2\eta^{ab} \frac{Re[x_{14} x_{25}]}{x_{12} x_{45}} \right) \\ &\times 2^{-1} (x_{14} x_{15} x_{24} x_{25})^{-1/2} (x_{45})^{-1/4} \end{aligned}$$

If we insert the above correlator into the S-matrix element, one can easily show that the amplitude is $SL(2, R)$ invariant. We do gauge fixing by setting $(x_1, x_2, z, \bar{z}) = (x, -x, i, -i)$ and taking $u = -\frac{\alpha'}{2}(k_1 + k_2)^2$, the final outcome for the amplitude is ⁵

$$\begin{aligned} \mathcal{A}^{C^{-2}T^0T^0} &\sim 16i \text{Tr}(\lambda_1 \lambda_2) k_{1a} k_{2b} \int_{-\infty}^{\infty} dx (2x)^{-2u-1} \left((1+x^2) \right)^{2u} \\ &\times \left[\text{Tr}(P_- \mathcal{C}'_{(n-1)} M_p \Gamma^{ba}) - 2\eta^{ab} \frac{1-x^2}{4xi} \text{Tr}(P_- \mathcal{C}'_{(n-1)} M_p) \right], \end{aligned}$$

obviously the second term in above does not have any contribution to the entire S-matrix and the ultimate result reads off as

$$\mathcal{A}^{C^{-2}T^0T^0} = \frac{i\mu_p}{4} \text{Tr}(\lambda_1 \lambda_2) 2\pi k_{1a} k_{2b} \frac{\Gamma(-2u)}{\Gamma(1/2-u)^2} \text{Tr}(P_- \mathcal{C}'_{(n-1)} M_p \Gamma^{ba}) \quad (15)$$

meanwhile this amplitude in symmetric picture has been done in [26] as follows

$$\mathcal{A}^{C^{-1}T^{-1}T^0} \sim \text{Tr}(\lambda_1 \lambda_2) 2\pi \frac{\Gamma(-2u)}{\Gamma(1/2-u)^2} \text{Tr}(P_- \mathcal{H}_{(n)} M_p \gamma^a) k_{2a} \quad (16)$$

If we use the momentum conservation as $-k_1^a - p^a = k_2^a$ and more crucially apply the following Bianchi identity $p_a \epsilon^{a_0 \dots a_{p-1} a} = 0$, then we get to know that the S-matrix is antisymmetric under interchanging tachyons. Note that to make sense of the amplitude in asymmetric picture, $p_a \epsilon^{a_0 \dots a_{p-2} ba}$ should be non-vanishing.

The amplitude is normalized by $(i\mu_p/4)$ where β is WZ constant and μ_p becomes brane's RR charge. The trace can be extracted as

$$\text{Tr} \left(\mathcal{C}'_{(n-1)} M_p (\Gamma^{ba}) \right) \delta_{p,n} = \pm \frac{32}{(p-1)!} \epsilon^{a_0 \dots a_{p-2} ba} C_{a_0 \dots a_{p-2}} \delta_{p,n}$$

⁵ $\alpha' = 2$ is set.

The trace including γ^{11} confirms to us the fact that all the above results keep working for the following as well

$$p > 3, H_n = *H_{10-n}, n \geq 5.$$

It is discussed in [43] that by sending either $k_i.k_j \rightarrow 0$ or $(k_i + k_j)^2 \rightarrow 0$, one finds out all massless or even tachyon singularities of BPS or non-BPS S-matrices. We know that there is a non-zero coupling between $C_{p-1} \wedge F$, two tachyons and a gauge field coupling, hence the correct momentum expansion for this amplitude is

$$u = -p^a p_a \rightarrow 0. \quad (17)$$

which is consistent with the fact that $p^a p_a$ has to be sent to zero for brane -anti brane systems [27], while for non- BPS branes the constraint gets replaced by $-p^a p_a \rightarrow \frac{-1}{4}$.

Let us consider the expansion of the pre-factor in string amplitude as

$$2\pi \frac{\Gamma(-2u)}{\Gamma(1/2 - u)^2} = -\frac{1}{u} + \sum_{m=-1}^{\infty} c_m(u)^{m+1}. \quad (18)$$

with the following coefficients

$$\begin{aligned} c_{-1} &= 4\ln(2), c_0 = \left(\frac{\pi^2}{6} - 8\ln(2)^2\right), \\ c_1 &= \frac{2}{3}(3\zeta(3) - \pi^2\ln(2) + 16\ln(2)^3) \end{aligned}$$

now we just want to produce the only massless gauge pole of this four point function by the following sub-amplitude

$$\mathcal{A} = V_a(C_{p-1}, A^{(1)})G_{ab}(A)V_b(A^{(1)}, T_1, T_2) + V_a(C_{p-1}, A^{(2)})G_{ab}(A)V_b(A^{(2)}, T_1, T_2) \quad (19)$$

First we need to take into account the following Chern-Simons coupling on the world volume of brane anti-brane system

$$i\mu_p(2\pi\alpha') \int_{\Sigma_{p+1}} \epsilon^{a_0 \dots a_p} \left(\text{Tr} (C_{a_0 \dots a_{p-2}} d_{a_{p-1}} (A_{1a_p} - A_{2a_p})) \right), \quad (20)$$

Note that the off-shell gauge field has to be $A^{(1)}$ and $A^{(2)}$ to actually get the consistent result so all the field theory vertices and propagators are

$$G_{ab}(A) = \frac{i\delta_{ab}}{(2\pi\alpha')^2 T_p(k^2)}$$

$$\begin{aligned}
V_b(A^{(1)}, T_1, T_2) &= iT_p(2\pi\alpha')(k_1 - k_2)_b \\
V_b(A^{(2)}, T_1, T_2) &= -iT_p(2\pi\alpha')(k_1 - k_2)_b \\
V_a(C_{p-1}, A^{(1)}) &= i\mu_p(2\pi\alpha') \frac{1}{(p-1)!} \epsilon_{a_0 \dots a_{p-1} a} C^{a_0 \dots a_{p-2}} k^{a_{p-1}} \\
V_a(C_{p-1}, A^{(2)}) &= -i\mu_p(2\pi\alpha') \frac{1}{(p-1)!} \epsilon_{a_0 \dots a_{p-1} a} C^{a_0 \dots a_{p-2}} k^{a_{p-1}}
\end{aligned} \tag{21}$$

Considering the momentum conservation $k^a = (k_1 + k_2)^a = -p^a$ and substituting them in (19), we actually obtain precisely the same string elements in the field theory amplitude as follows

$$\mathcal{A} = 4i\mu_p \frac{1}{(p-1)!u} \epsilon^{a_0 \dots a_{p-2} ab} C_{a_0 \dots a_{p-2}} k_{1a} k_{2b} \tag{22}$$

Lets generate all the infinite contact interactions.

If we start expanding out all the Gamma functions inside the amplitude, we then derive all infinite α' higher derivative corrections to two real tachyons of brane anti brane system and a C_{p-1} potential field. Consider the following coupling

$$2i\beta^2 \mu_p (2\pi\alpha')^2 \int_{\Sigma_{p+1}} \left(\text{Tr} (C_{p-1} \wedge DT \wedge DT^*) \right), \tag{23}$$

now all order α' higher derivative corrections to (23) are derived by taking into account $u \rightarrow 0$ in the amplitude.

Replacing the expansion to S-matrix and making the comparisons with EFT coupling (23), we believe that c_{-1} term is regenerated by (23) so that the first contact term of string amplitude is

$$i\mu_p 16 \ln 2 \frac{1}{(p-1)!} \epsilon^{a_0 \dots a_{p-2} ba} C_{a_0 \dots a_{p-2}} k_{1a} k_{2b} \tag{24}$$

if we compare (24) with (23) then we can realise that, this fixes the normalisation of WZ as $\beta = \frac{1}{\pi} \sqrt{\frac{2 \ln(2)}{\alpha'}}$ and the second contact interaction becomes

$$i\mu_p 4u \left(\frac{\pi^2}{6} - 8 \ln(2)^2 \right) \frac{1}{(p-1)!} \epsilon^{a_0 \dots a_{p-2} ba} C_{a_0 \dots a_{p-2}} k_{1a} k_{2b} \tag{25}$$

where this can be produced by the following α' higher derivative corrections as

$$i(\alpha')^2 \mu_p \left(\frac{\pi^2}{6} - 8 \ln(2)^2 \right) \epsilon^{a_0 \dots a_p} C_{a_0 \dots a_{p-2}} (D^b D_b) D_{a_{p-1}} T D_{a_p} T^* \tag{26}$$

One can keep finding out all the other higher derivative corrections of (23) so that all order α' corrections of one RR potential field and two real tachyons of brane anti brane system within compact formula can be explored as follows

$$i\mu_p(2\pi\alpha')^2 C_{(p-1)} \wedge \text{Tr} \left(\sum_{m=-1}^{\infty} c_m(\alpha'(D^b D_b))^{m+1} DT \wedge DT^* \right) \quad (27)$$

notice that due to the constraint of producing the correct momentum expansion of brane anti brane system which is $u = -p^a p_a \rightarrow 0$, the following corrections

$$i\mu_p(2\pi\alpha')^2 C_{(p-1)} \wedge \text{Tr} \left(\sum_{m=-1}^{\infty} c_m(\alpha')^{m+1} D_{a_1} \cdots D_{a_{m+1}} DT \wedge D^{a_1} \cdots D^{a_{m+1}} DT^* \right) \quad (28)$$

cannot be entirely worked out and indeed the only exact and correct all order α' corrections of this system is indeed (27), it is worth highlighting the essential point as follows. We were looking for two tachyons and a RR potential C field in the world volume of brane anti brane systems, therefore in above corrections all commutator terms must be overlooked.

3 All order $< C^{-2} A^0 T^0 T^0 >$ S-Matrix

In this section we would like to actually carry out the entire S-matrix of a potential C-field, one gauge field and two real tachyons in the world volume of brane anti brane system to compare it with the same S-matrix in symmetric picture and eventually see whether or not one needs to add extra potential terms (just in the presence of world volume fields such as gauge field and tachyons) to the vertex operator of RR in asymmetric picture. All the vertices in the presence of brane anti brane system within their CP factor can be constructed as

$$\begin{aligned} V_T^{(0)}(x) &= \alpha' i k \cdot \psi(x) e^{\alpha' i k \cdot X(x)} \lambda \otimes \sigma_1, \\ V_T^{(-1)}(x) &= e^{-\phi(x)} e^{\alpha' i k \cdot X(x)} \lambda \otimes \sigma_2 \\ V_A^{(-1)}(x) &= e^{-\phi(x)} \xi_a \psi^a(x) e^{\alpha' i q X(x)} \lambda \otimes \sigma_3 \\ V_A^{(0)}(x) &= \xi_{1a} (\partial^a X(x) + i \alpha' k \cdot \psi \psi^a(x)) e^{\alpha' i k \cdot X(x)} \otimes I \\ V_C^{(-\frac{3}{2}, -\frac{1}{2})}(z, \bar{z}) &= (P_- \mathcal{C}'_{(n-1)} M_p)^{\alpha\beta} e^{-3\phi(z)/2} S_\alpha(z) e^{i \frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i \frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \lambda \otimes I \end{aligned} \quad (29)$$

This S-matrix is given by

$$\mathcal{A}^{C^{-2} A^0 T^0 T^0} \sim \int dx_1 dx_2 dx_3 dz d\bar{z} \langle V_A^{(0)}(x_1) V_T^{(0)}(x_2) V_T^{(0)}(x_3) V_{RR}^{(-\frac{3}{2}, -\frac{1}{2})}(z, \bar{z}) \rangle, \quad (30)$$

The entire amplitude can be divided out to two separate correlation functions. We also need to make use of the generalized Wick-like rule. Define

$$s = \frac{-\alpha'}{2}(k_1 + k_3)^2, \quad t = \frac{-\alpha'}{2}(k_1 + k_2)^2, \quad u = \frac{-\alpha'}{2}(k_2 + k_3)^2$$

It is easy to find out all the correlators and show that the amplitude is $SL(2, \mathbb{R})$ invariant. Note that we did gauge fixing as $x_1 = 0, x_2 = 1, x_3 = \infty$ so that the amplitude after gauge fixing is written down as follows

$$\begin{aligned} \mathcal{A}^{C^{-2}A^0T^0T^0} &\sim \int dx_4 dx_5 (P_- \mathcal{C}'_{(n-1)} M_p)^{\alpha\beta} \xi_{1a} (-4k_{2b} k_{3c}) x_{45}^{-2(t+s+u+1)} |x_4|^{2t+2s+1} |1-x_4|^{2t+2u-1/2} \\ &\times \left[\left((\Gamma^{cb} C^{-1})_{\alpha\beta} + 2\eta^{bc} (C^{-1})_{\alpha\beta} \left(\frac{Re[1-x_4]}{x_{45}} \right) \right) I_{11} + ik_{1d} |x_4|^{-2} x_{45} I_{22} \right] \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \end{aligned}$$

where

$$\begin{aligned} I_{11} &= -i|x_4|^{-2}(x_4 + x_5)(k_2^a + k_3^a) + 2ik_2^a, \\ I_{22} &= \left[(\Gamma^{cbad} C^{-1})_{\alpha\beta} + 2 \left(\eta^{db} (\Gamma^{ca} C^{-1})_{\alpha\beta} - \eta^{ab} (\Gamma^{cd} C^{-1})_{\alpha\beta} \right) \left(\frac{x - |x_4|^2}{x_{45}} \right) \right. \\ &\quad + 2 \left(-\eta^{dc} (\Gamma^{ba} C^{-1})_{\alpha\beta} + \eta^{ac} (\Gamma^{bd} C^{-1})_{\alpha\beta} \right) \frac{x}{x_{45}} \\ &\quad \left. + 2 \left(\eta^{bc} (\Gamma^{ad} C^{-1})_{\alpha\beta} \right) \left(\frac{1-x}{x_{45}} \right) + 4(-\eta^{db} \eta^{ac} + \eta^{dc} \eta^{ab}) \left(\frac{x^2 - x|x_4|^2}{(x_{45})^2} \right) \right] \end{aligned}$$

All the integrations on upper half plane can be evaluated on the location of closed string and the final result can be written down just in terms of four Gamma functions over three Gamma functions by using the important results of the following integrals

$$\int d^2z |1-z|^a |z|^b (z-\bar{z})^c (z+\bar{z})^d$$

that are accommodated for $d = 0, 1$ in [42] and for $d = 2$ in [43] where $z = x_4 = x + iy, \bar{z} = x_5 = x - iy, x_{ij} = x_i - x_j$.

First of all let us see what happens to the terms that are apparently absent in symmetric picture. Hence, we are now considering the terms in asymmetric amplitude that after all just involve $\text{Tr}(P_- \mathcal{C}'_{(n-1)} M_p)$ as follows:

$$\mathcal{A}_3^{C^{-2}A^0T^0T^0} = -4i I_{33} \xi_{1a} (2)^{-2(t+s+u)-3} \pi \frac{\Gamma(-u) \Gamma(-s - \frac{3}{4}) \Gamma(-t + \frac{1}{4}) \Gamma(-t - s - u - 1)}{\Gamma(-u - t + \frac{1}{4}) \Gamma(-t - s + \frac{1}{2}) \Gamma(-s - u + \frac{1}{4})}$$

where

$$\begin{aligned}
I_{33} = & 2(u + \frac{1}{2})(k_2^a + k_3^a)(-t + \frac{1}{4})(-s - u - \frac{3}{4}) \\
& - 2(u + \frac{1}{2})(k_2^a)(-t - s - \frac{1}{2})(-s - u - \frac{3}{4}) \\
& - 2(u + \frac{1}{2})(k_2^a + k_3^a)(\frac{1}{2}(-s - \frac{3}{4}) - u(-t + \frac{1}{4})) \\
& + 2(u + \frac{1}{2})(k_2^a)(-t - s - \frac{1}{2})(-u) \\
& + (\frac{1}{2}(-s - \frac{3}{4}) - u(-t + \frac{1}{4}))(2(t + \frac{1}{4})k_3^a - 2(s + \frac{1}{4})k_2^a) \\
& + u(-t - s - \frac{1}{2})(2(t + \frac{1}{4})k_3^a - 2(s + \frac{1}{4})k_2^a)
\end{aligned} \tag{31}$$

One might think that these terms are needed in the entire S-matrix, however, after some simplification, one can readily see that the sum of coefficients of k_2^a and k_3^a separately vanishes so that the whole $\mathcal{A}_3^{C^{-2}A^0T^0T^0}$ vanishes and has no contribution to the S-matrix at all.

Eventually after having performed massive computations on all the integrations and making use of momentum conservation along the world volume of brane, one is able to get the final form of S-matrix in asymmetric picture as follows

$$\mathcal{A}^{C^{-2}A^0T^0T^0} = \mathcal{A}_1 + \mathcal{A}_2 \tag{32}$$

where

$$\begin{aligned}
\mathcal{A}_1 & \sim -4i\xi_{1a}k_{1d}k_{2b}k_{3c}\text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \Gamma^{cbad}) L_1, \\
\mathcal{A}_2 & \sim -4ip_d \text{Tr}(P_- \mathcal{C}_{(n-1)} M_p \Gamma^{ad}) \left(2k_2 \cdot \xi_1 k_3^a (s + \frac{1}{4}) + 2k_3 \cdot \xi_1 k_2^a (t + \frac{1}{4}) - \xi_{1a} (t + \frac{1}{4})(s + \frac{1}{4}) \right) L_2
\end{aligned}$$

with

$$\begin{aligned}
L_1 & = (2)^{-2(t+s+u)-1} \pi \frac{\Gamma(-u)\Gamma(-s+\frac{1}{4})\Gamma(-t+\frac{1}{4})\Gamma(-t-s-u)}{\Gamma(-u-t+\frac{1}{4})\Gamma(-t-s+\frac{1}{2})\Gamma(-s-u+\frac{1}{4})}, \\
L_2 & = (2)^{-2(t+s+u+1)} \pi \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s-\frac{1}{4})\Gamma(-t-\frac{1}{4})\Gamma(-t-s-u-\frac{1}{2})}{\Gamma(-u-t+\frac{1}{4})\Gamma(-t-s+\frac{1}{2})\Gamma(-s-u+\frac{1}{4})},
\end{aligned}$$

Just for the record we write down the results in all different symmetric picture (in terms of RR's field strength not its potential any more) as follows:

$$\begin{aligned}
\mathcal{A}^{C^{-1}A^0T^{-1}T^0} &= \mathcal{A}^{C^{-1}A^{-1}T^0T^0} = \frac{i\mu_p}{2\sqrt{\pi}} \left[k_{3c}k_{2b}\xi_{1a} \text{Tr} \left((P_- \mathbb{H}_{(n)} M_p \Gamma^{cba}) \right) L_1 + \text{Tr} \left((P_- \mathbb{H}_{(n)} M_p) \gamma^a \right) L_2 \right. \\
&\quad \left. \times \left\{ k_{2a}(t+1/4)(2\xi_{1\cdot}k_3) + k_{3a}(s+1/4)(2\xi_{1\cdot}k_2) - \xi_{1a}(s+1/4)(t+1/4) \right\} \right] \quad (33)
\end{aligned}$$

Now if we use momentum conservation along the world volume of brane as $(k_1 + k_2 + k_3)^d = -p^d$ and use the fact that \mathcal{A}_1 of (32) is symmetric with respect to k_2, k_3 but antisymmetric under $\epsilon^{a_0 \dots a p - 3 b c d}$, therefore the first term of asymmetric picture does match with the 1st term of symmetric amplitude and likewise for the other terms, where $p_d \mathcal{C}_{(n-1)} = \mathbb{H}_{(n)}$ has been used.

This evidently confirms that the vertex operator of RR in asymmetric picture is exact and complete. It also confirms that no extra potential terms needed to be added to RR potential vertex operator in asymmetric picture. It also reveals that for the mixed world volume S-matrices of closed string RR, gauge fields and tachyons, even in five and higher point functions there is no picture dependence at all. However, the story gets complicated for the higher point functions of the mixed closed string RR and scalar fields as we will point out in detail in the next section.

4 All order Bulk Singularities of $\langle C^{(-2)}\phi^{(0)}T^{(0)}T^{(0)} \rangle$ S-matrix in Brane Anti Brane System

The general structures of vertex operators with their Chan-Paton factors in brane anti brane system can be shown by

$$\begin{aligned}
V_\phi^{(0)}(x) &= \xi_{1i}(\partial X^i(x) + i\alpha' k \cdot \psi \psi^i(x)) e^{\alpha' i k \cdot X(x)} \otimes I \\
V_\phi^{(-1)}(x) &= e^{-\phi(x)} \xi_i \psi^i(x) e^{\alpha' i q \cdot X(x)} \lambda \otimes \sigma_3 \\
V_\phi^{(-2)}(x) &= e^{-2\phi(x)} V_\phi^{(0)}(x) \\
V_C^{(-\frac{1}{2}, -\frac{1}{2})}(z, \bar{z}) &= (P_- \mathbb{H}_{(n)} M_p)^{\alpha\beta} e^{-\phi(z)/2} S_\alpha(z) e^{i\frac{\alpha'}{2} p \cdot X(z)} e^{-\phi(\bar{z})/2} S_\beta(\bar{z}) e^{i\frac{\alpha'}{2} p \cdot D \cdot X(\bar{z})} \lambda \otimes \sigma_3
\end{aligned}$$

The ultimate form of a field strength RR, one transverse scalar field and two real tachyons in the following picture is derived in [27] to be

$$\mathcal{A}^{C^{(-1)}\phi^{(-1)}T^{(0)}T^{(0)}} = \mathcal{A}_1 + \mathcal{A}_2 \quad (34)$$

where

$$\begin{aligned}\mathcal{A}_1 &\sim -8\xi_{1i}k_{2a}k_{3b}2^{-3/2}\text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{bai}), \\ \mathcal{A}_2 &\sim 8\xi_{1i}2^{-3/2}\text{Tr}(P_- \not{H}_{(n)} M_p \gamma^i)(t + \frac{1}{4})(s + \frac{1}{4})L_2\end{aligned}\quad (35)$$

where

L_1, L_2 are already given in the last section.

Note that since there is a non zero correlation function of $\langle e^{ip \cdot X(z)}(\partial X^i(x_1)) \rangle$ and more importantly, there is no Ward identity for mixed closed RR and transverse scalar fields, one needs to think of all the bulk singularities that do include an infinite number of $p \cdot \xi$ terms as well as keep an eye on asymmetric picture for which we are going to illustrate on that right now. Note that in asymmetric amplitude, we now get some extra singularities in the bulk of brane anti brane system that carry $p \cdot \xi$ terms.

Indeed by dealing with the same elements and coming over its last symmetric change of picture or for the other symmetric result of $\langle C^{(-1)}\phi^{(0)}T^{(-1)}T^{(0)} \rangle$ we get the final answer in the presence of brane anti brane as below

$$\mathcal{A}^{C^{(-1)}\phi^{(0)}T^{(-1)}T^{(0)}} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 \quad (36)$$

so that this turn their part gets replaced by

$$\begin{aligned}\mathcal{A}_1 &\sim -2^{3/2}ip^i\xi_{1i}k_{3b}\text{Tr}(P_- \not{H}_{(n)} M_p \gamma^b)L_1, \\ \mathcal{A}_2 &\sim -2^{3/2}i\xi_{1i}\left\{\text{Tr}(P_- \not{H}_{(n)} M_p \Gamma^{bia})\right\}k_{1a}k_{3b}L_1 \\ \mathcal{A}_3 &\sim -2^{3/2}i\xi_{1i}\left\{\text{Tr}(P_- \not{H}_{(n)} M_p \gamma^i)\right\}(t + \frac{1}{4})(s + \frac{1}{4})L_2\end{aligned}\quad (37)$$

Note that one might make use of momentum conservation along the world volume of brane and use some sort of generalized Bianchi identities ⁶ to get rid of the first term of (37), however, this did happen if and only if we would have not been able to produce those infinite u-channel Bulk singularities that carry $p \cdot \xi$ term in an Effective Field Theory. Let us further elaborate on that by taking into account the asymmetric result as well.

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$$\xi_{1i}k_{3b}\left(-p_a\epsilon^{a_0\cdots a_{p-2}ab}H_{a_0\cdots a_{p-2}}^i + p^i\epsilon^{a_0\cdots a_{p-1}b}H_{a_0\cdots a_{p-1}}\right) = 0 \quad (38)$$

Indeed it is easy to find out all the correlators of a C-field in asymmetric picture and a real transverse scalar field and two real tachyons of brane anti brane system of $< C^{(-2)}\phi^{(0)}T^{(0)}T^{(0)} >$ and show that the amplitude is $SL(2,R)$ invariant. Note that we did gauge fixing as $x_1 = 0, x_2 = 1, x_3 = \infty$ so that the amplitude after gauge fixing is written down as follows

$$\begin{aligned} \mathcal{A}^{C^{-2}\phi^0T^0T^0} &\sim \int dx_4 dx_5 (P_- \mathcal{C}'_{(n-1)} M_p)^{\alpha\beta} \xi_{1i} (-4k_{2b}k_{3c}) x_{45}^{-2(t+s+u)-1} |x_4|^{2t+2s-1} |1-x_4|^{2t+2u-1/2} \\ &\times \left[-ip^i \left((\Gamma^{cb}C^{-1})_{\alpha\beta} + 2\eta^{bc}(C^{-1})_{\alpha\beta} \left(\frac{Re[1-x_4]}{x_{45}} \right) \right) + ik_{1a}I_{44} \right] \text{Tr}(\lambda_1\lambda_2\lambda_3) \end{aligned}$$

where

$$\begin{aligned} I_{44} &= \left[(\Gamma^{cbia}C^{-1})_{\alpha\beta} + 2 \left(\eta^{ab}(\Gamma^{ci}C^{-1})_{\alpha\beta} \right) \left(\frac{x - |x_4|^2}{x_{45}} \right) \right. \\ &\quad \left. - 2 \left(\eta^{ac}(\Gamma^{bi}C^{-1})_{\alpha\beta} \right) \frac{x}{x_{45}} + 2 \left(\eta^{bc}(\Gamma^{ia}C^{-1})_{\alpha\beta} \right) \left(\frac{1-x}{x_{45}} \right) \right] \end{aligned}$$

Taking the integrations on upper half plane and doing more simplifications, one gets the essential form of the amplitude, where more ingredients can be found from [37] so that

$$\mathcal{A}^{C^{-2}\phi^0T^0T^0} = \mathcal{A}_1 + \mathcal{A}_2 + \mathcal{A}_3 + \mathcal{A}_4 \quad (39)$$

where

$$\begin{aligned} \mathcal{A}_1 &\sim ip^i \xi_{1i} (4k_{2b}k_{3c}) \text{Tr}(P_- \mathcal{C}'_{(n-1)} M_p \Gamma^{cb}) L_1, \\ \mathcal{A}_2 &\sim 4ip^i \xi_{1i} \left\{ \text{Tr}(P_- \mathcal{C}'_{(n-1)} M_p) \right\} \left(t + \frac{1}{4} \right) \left(s + \frac{1}{4} \right) L_2 \\ \mathcal{A}_3 &\sim -4i \xi_{1i} k_{1a} k_{2b} k_{3c} \left\{ \text{Tr}(P_- \mathcal{C}'_{(n-1)} M_p \Gamma^{cbia}) \right\} L_1 \\ \mathcal{A}_4 &\sim 4i \xi_{1i} \left\{ \text{Tr}(P_- \mathcal{C}'_{(n-1)} M_p \Gamma^{bi}) \right\} \left(t + \frac{1}{4} \right) \left(s + \frac{1}{4} \right) L_2 (k_{1b} + k_{2b} + k_{3b}) \end{aligned} \quad (40)$$

notice that in this asymmetric result for mixed closed string RR in the presence of two real tachyons of brane anti brane and a transverse scalar field we have got four terms where two of them, namely, \mathcal{A}_1 and \mathcal{A}_2 are extra bulk singularities that have just been shown up in asymmetric result as have already been pointed out. These terms carry the scalar product of RR momentum and scalar polarization which are both located in the bulk, that is why we call them bulk singularity structures and in fact L_1 does have infinite u-channel singularities where as L_2 carries an infinite number of $(t' + s' + u)$ channel singularities where $s' = s + 1/4, t' = t + 1/4$ have been defined.

All (p,ξ) 's terms do have potentially several things to do with worthwhile issues within the field of perturbative string theory that has been going on over last few years and started by series of papers by Witten [44], also bulk terms can have different meanings if one starts to think about still non trivial moduli spaces [45].

One might consider the momentum conservation $(k_1 + k_2 + k_3)^a = -p^a$ to actually apply the Bianchi identity that has been used for BPS amplitudes ($p^b \epsilon^{a_0 \dots a_{p-1} b} = 0$) to this brane anti brane S-matrix , however, we claim that not all the Bianchi identities of BPS cases can hold for non-BPS, nor for brane anti brane amplitudes. The reasons for that conclusion is as follows. First of all we do need to keep track of those bulk singularities in the S-Matrix, secondly, not all the equations of BPS cases can be true for non-BPS or non-supersymmetric S-Matrices , indeed all the equations of BPS cases are so manifest, meanwhile this might not occur to non-BPS cases as things get changed after symmetry breaking happened. Hence for non-BPS branes, one needs to break certain several equations of BPS branes.

We already knew from EFT that we must have an infinite number of u-channel gauge field poles, therefore we no longer can add up the 3rd term of (40) with the 1st term of (40) to get generalized Bianchi identity. Indeed the same story holds for the other terms in the above amplitude. Basically all the terms appearing in this amplitude, are needed in an EFT formalism as we start producing all those new couplings in the next section.

5 All order u- channel Bulk Singularity Structures of $< C^{-2} \phi^0 T^0 T^0 >$

It has been comprehensively explained in [27] how to explore the momentum expansions of tachyons, nevertheless we just hint that , by applying momentum conservation we get the constraint as $s' + t' + u = -p^a p_a$ where $s' = s + \frac{1}{4}$, $t' = t + \frac{1}{4}$ and as highlighted in the last section for brane anti-brane system the soft condition $p^a p_a \rightarrow 0$ holds, thus the expansion is $u \rightarrow 0, s \rightarrow \frac{-1}{4}, t \rightarrow \frac{-1}{4}$. In order to deal with u-channel gauge poles, we need to first employ \mathcal{A}_1 and eventually \mathcal{A}_3 , extract the traces and consider all order α' expansion of L_1 as below

$$(\mathcal{A}_1)^{C^{-2} \phi^0 T^0 T^0} = \epsilon^{a_0 \dots a_{p-2} c b} p^i C_{a_0 \dots a_{p-2}} \frac{64}{(p-1)!} L_1 \xi_{1i} k_{3c} k_{2b} \quad (41)$$

Indeed one can see that some terms of L_1 expansion have the same behaviour as we

have seen in the case of $< C^{-2}T^0T^0 >$ S-matrix , as

$$L_1 = \pi^{3/2} \left(\frac{-1}{u} + 4\ln(2) + \left(\frac{\pi^2}{6} - 8\ln(2)^2 \right) u - \frac{\pi^2}{6} \frac{(s' + t')^2}{u} + \dots \right) \quad (42)$$

Given the fact that there are prescriptions that one can investigate all order α' corrections of string amplitudes, we just write down the compact and all order expansion of L_1 that has been achieved in [27]

$$L_1 = \pi^{3/2} \left(-\frac{1}{u} \sum_{n=-1}^{\infty} b_n (s' + t')^{n+1} + \sum_{p,n,m=0}^{\infty} f_{p,n,m} u^p (s' t')^n (s' + t')^m \right) \quad (43)$$

with some of its own coefficients

$$\begin{aligned} b_{-1} &= 1, \quad b_0 = 0, \quad b_1 = \frac{\pi^2}{6}, \quad b_2 = 2\zeta(3), \quad a_0 = 4\ln 2, \\ a_1 &= \frac{\pi^2}{6} - 8\ln(2)^2, \quad a_2 = \frac{2}{3}(-\pi^2 \ln 2 + 3\zeta(3) + 16\ln(2)^3), \\ f_{0,0,2} &= \frac{2}{3}\pi^2 \ln(2), \quad f_{0,1,0} = -14\zeta(3), \quad f_{0,0,3} = 8\zeta(3) \ln(2), \end{aligned} \quad (44)$$

In order to actually produce all infinite u- channel bulk singularities, one first needs to consider the following Feynman rule

$$\mathcal{A} = V_a(C_{p-1}, \phi, A) G_{ab}(A) V_b(A, T_1, T_2) \quad (45)$$

with the following EFT vertex and propagator

$$\begin{aligned} G_{ab}(A) &= \frac{i\delta_{ab}}{(2\pi\alpha')^2 T_p k^2} \\ V_b(A, T_1, T_2) &= T_p (2\pi\alpha') (k_{2b} - k_{3b}) \end{aligned} \quad (46)$$

where $k^2 = \frac{\alpha'}{2}(k_2 + k_3)^2 = -u$ and the kinetic term of gauge field $\text{Tr}(\frac{-1}{4}F_{ab}F^{ba})$ has been taken. $V_b(A, T_1, T_2)$ is derived from kinetic term of Tachyons in DBI action $\text{Tr}(2\pi\alpha' D_a T D^a T)$. Notice that all kinetic terms of gauge field, scalar and tachyons have been already fixed, so that they do not receive any correction terms at all.

Now consider the mixing Chern-Simons coupling as well as Taylor expanded of scalar field to get to $V_a(C_{p-1}, \phi, A)$ as follows

$$\mu_p (2\pi\alpha')^2 \int_{\sum_{(p+1)}} \text{Tr} \left(\partial_i C_{p-1} \wedge F \phi^i \right) \quad (47)$$

Particular attention should be drawn to the fact that we do not need to take integration by parts, having set (47), we get to

$$V_a(C_{p-1}, \phi, A) = \mu_p(2\pi\alpha')^2 \frac{1}{(p-1)!} \epsilon^{a_0 \dots a_{p-2}} p^i C_{a_0 \dots a_{p-2}} k_a \xi_{1i} \quad (48)$$

where k is the momentum of off-shell gauge field, $k_a = -(k_2 + k_3)_a$. If we do normalise the string amplitude by $\frac{i\mu_p}{8\sqrt{\pi}}$ and replace (48) and (46) inside (45) then we are precisely able to produce the first u-channel gauge pole of (41) and (42) in an Effective field theory as well.

Notice that the second and third terms of (42) do relate to contact terms and both can be produced in EFT by taking the following couplings accordingly. ⁷

As we can see from the expansion of L_1 , it has so many singularities and in order to produce them, one needs to apply proper all order α' corrections to Chern-Simons coupling, because the kinetic terms of gauge field and tachyons do not gain any correction as they have been fixed in DBI action. Let us apply all order α' higher derivative corrections as

$$\mu_p(2\pi\alpha')^2 \int_{\sum_{(p+1)}} \partial_i C_{p-1} \wedge \sum_{n=-1}^{\infty} b_n(\alpha')^{n+1} \text{Tr} \left(D_{a_1} \dots D_{a_{n+1}} F D^{a_1} \dots D^{a_{n+1}} \phi^i \right) \quad (50)$$

to be able to obtain all order vertex of $V_a(C_{p-1}, \phi, A)$ as follows

$$V_a(C_{p-1}, \phi, A) = \mu_p(2\pi\alpha')^2 \frac{1}{(p-1)!} \epsilon^{a_0 \dots a_{p-2}} p^i C_{a_0 \dots a_{p-2}} (k_2 + k_3)^a \xi_i \sum_{n=-1}^{\infty} b_n(\alpha' k_1 \cdot k)^n \quad (51)$$

substituting (51) and (46) inside (45), we are precisely able to produce all order u-channel gauge poles of (41) in an Effective field theory as below

$$\mathcal{A} = \mu_p(2\pi\alpha') \frac{2i}{(p-1)!u} \epsilon^{a_0 \dots a_{p-2} bc} p^i C_{a_0 \dots a_{p-2}} k_{2b} k_{3c} \xi_{1i} \sum_{n=-1}^{\infty} b_n \left(\frac{\alpha'}{2} \right)^{n+1} (s' + t')^{n+1} \quad (52)$$

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$$\begin{aligned} & \frac{i}{2} \mu_p \beta^2 (2\pi\alpha')^3 \text{Tr} \left(\partial_i C_{p-1} \wedge DT \wedge DT^* (\phi^1 + \phi^2)^i \right) \\ & \frac{i}{2} \mu_p (2\pi\alpha') (\alpha')^2 \left(\frac{\pi^2}{6} - 8 \ln 2^2 \right) \text{Tr} \left(\partial_i C_{p-1} \wedge D^a D_a (DT \wedge DT^*) (\phi^1 + \phi^2)^i \right) \end{aligned} \quad (49)$$

Note that all order contact interactions of this S-matrix can be found in section 3.1 of [27]. Let us produce another all infinite bulk singularities of \mathcal{A}_3 . Thus we extract the traces and consider all order α' expansion of L_1 as below

$$(\mathcal{A}_3)^{C^{-2}\phi^0 T^0 T^0} = -\epsilon^{a_0 \dots a_{p-3} abc} p_a C_{a_0 \dots a_{p-3}}^i \frac{64}{(p-1)!} L_1 \xi_{1i} k_{3c} k_{2b} \quad (53)$$

where we have used momentum conservation as well as the fact that the 3rd term of amplitude was symmetric under interchanging both k_2, k_3 but also is anti symmetric as it does involve ϵ tensor, therefore just p^a terms remain after applying momentum conservation to the 3rd term of S-matrix.

In order to actually produce the second kind of an infinite u- channel bulk singularities of \mathcal{A}_3 , one needs to consider the same Feynman rule as appeared in (45) where the same definitions for propagator and $V_b(A, T_1, T_2)$ kept held. But one has to define a new sort of coupling as follows

$$\frac{1}{(p-1)!} \mu_p (2\pi\alpha')^2 \int_{\sum_{(p+1)}} \text{Tr} \left(C_{a_0 \dots a_{p-3}}^i F_{a_{p-2} a_{p-1}} D_{a_p} \phi^i \right) \epsilon^{a_0 \dots a_p} \quad (54)$$

where in (54) the scalar field comes from pull-back of brane and the Chern-Simons coupling also has been taken into account. As we discussed there are infinite singularities and in order to reconstruct them out, one needs to apply all order α' higher derivative corrections to the new coupling (54) as follows

$$\frac{\mu_p (2\pi\alpha')^2}{(p-1)!} \int_{\sum_{(p+1)}} \text{Tr} \left(C_{a_0 \dots a_{p-3}}^i \sum_{n=-1}^{\infty} b_n (\alpha')^{n+1} D_{a_1} \dots D_{a_{n+1}} F_{a_{p-2} a_{p-1}} D^{a_1} \dots D^{a_{n+1}} D_{a_p} \phi^i \right) \quad (55)$$

to be able to obtain all order vertex of $V_a(C_{p-1}, \phi, A)$ as follows

$$\begin{aligned} V_a(C_{p-1}, \phi, A) &= \mu_p (2\pi\alpha')^2 \frac{1}{(p-1)!} \epsilon^{a_0 \dots a_{p-1}} p_{a_{p-1}} C_{a_0 \dots a_{p-3}}^i (k_2 + k_3)_{a_{p-2}} \xi_i \\ &\quad \times \sum_{n=-1}^{\infty} b_n (\alpha' k_1 \cdot k)^{n+1} \end{aligned} \quad (56)$$

Notice to the point that in (54), the derivative D_{a_p} can not act on $d_{a_{p-2}} A_{a_{p-1}}$ because it is symmetric under interchanging the derivatives but is antisymmetric under ϵ tensor so the result is zero, hence after taking integration by parts D_{a_p} can just act on $C_{a_0 \dots a_{p-3}}^i$.

Replacing (56) and (46) inside (45), we are precisely able to produce all order u-channel gauge poles of (53) in an effective field theory as below

$$\mathcal{A} = \mu_p(2\pi\alpha') \frac{2i}{(p-1)!u} \epsilon^{a_0 \dots a_{p-2} abc} p^a C_{a_0 \dots a_{p-3}}^i k_{2b} k_{3c} \xi_{1i} \sum_{n=-1}^{\infty} b_n \left(\frac{\alpha'}{2}\right)^{n+1} (s' + t')^{n+1} \quad (57)$$

Therefore by making use of the new coupling (54) we were able to exactly regenerate all infinite u-channel bulk singularities of this S-matrix.

6 Infinite $(u + s' + t')$ channel Bulk singularities

The amplitude is non zero for $n - 1 = p + 1$ case and it involves both \mathcal{A}_2 and \mathcal{A}_4 terms, also note that the amplitude for this case is symmetric under interchanging the tachyons, let us extract the traces and write them down explicitly as below

$$\begin{aligned} \mathcal{A}_2 &= \frac{8i\mu_p}{\sqrt{\pi}(p+1)!} p^i \xi_{1i} \epsilon^{a_0 \dots a_p} C_{a_0 \dots a_p} t' s' L_2 \\ \mathcal{A}_4 &= \frac{8i\mu_p}{\sqrt{\pi}(p+1)!} \xi_{1i} \epsilon^{a_0 \dots a_{p-1} b} C_{a_0 \dots a_{p-1}}^i t' s' L_2 (k_{1b} + k_{2b} + k_{3b}) \end{aligned} \quad (58)$$

Note that there is no Ward identity associated to the scalar field in the presence of RR. The closed form of the expansion of $t' s' L_2$ is given by

$$\begin{aligned} t' s' L_2 &= \frac{\sqrt{\pi}}{2} \left(\frac{-1}{(t' + s' + u)} + \sum_{n=0}^{\infty} a_n (s' + t' + u)^n + \frac{\sum_{n,m=0}^{\infty} l_{n,m} (s' + t')^n (t' s')^{m+1}}{(t' + s' + u)} \right. \\ &\quad \left. + \sum_{p,n,m=0}^{\infty} e_{p,n,m} (s' + t' + u)^p (s' + t')^n (t' s')^{m+1} \right) \end{aligned} \quad (59)$$

$l_{n,m}$ and $e_{p,n,m}$ are

$$\begin{aligned} l_{0,0} &= -\pi^2/3, & l_{1,0} &= 8\zeta(3) \\ l_{2,0} &= -7\pi^4/45, & l_{0,1} &= \pi^4/45, & l_{3,0} &= 32\zeta(5), & l_{1,1} &= -32\zeta(5) + 8\zeta(3)\pi^2/3 \\ e_{0,0,0} &= \frac{2}{3} (2\pi^2 \ln(2) - 21\zeta(3)), & e_{1,0,0} &= \frac{1}{9} (4\pi^4 - 504\zeta(3) \ln(2) + 24\pi^2 \ln(2)^2) \end{aligned} \quad (60)$$

Let us consider all order singularities related to \mathcal{A}_4 term, where we have applied momentum conservation along the world volume of brane and extracted the trace. We also take all the singularities in the expansion of $t' s' L_2$ and replace them in \mathcal{A}_4 term so that all poles are written down as follows

$$\mathcal{A}_4 = -\frac{4i\mu_p}{(p+1)!}\xi_{1i}\epsilon^{a_0\cdots a_{p-1}b}p^b C_{a_0\cdots a_{p-1}}^i \frac{\sum_{n,m=0}^{\infty} l_{n,m}(s'+t')^n(t's')^{m+1}}{(t'+s'+u)} \quad (61)$$

The Lagrangian for two scalar, two tachyon couplings for non-BPS brane was defined as

$$\begin{aligned} L(\phi, \phi, T, T) = & -2T_p(\pi\alpha')^3 S\text{Tr} \left(m^2 T^2 (D_a \phi^i D^a \phi_i) + \frac{\alpha'}{2} D^\alpha T D_\alpha T D_a \phi^i D^a \phi_i \right. \\ & \left. - \alpha' D^b T D^a T D_a \phi^i D_b \phi_i \right) \end{aligned} \quad (62)$$

Note that all order corrections to two tachyons and two scalar fields of brane anti brane system have been derived in [27], but for the completeness we just write them down because in order to produce an infinite singularities, we need to deal with them as well.

$$L = -2T_p(\pi\alpha')(\alpha')^{2+n+m} \sum_{n,m=0}^{\infty} (L_1^{nm} + L_2^{nm} + L_3^{nm} + L_4^{nm}), \quad (63)$$

where

$$\begin{aligned} L_1^{nm} &= m^2 \text{Tr} \left(a_{n,m} [D_{nm} (TT^* D_a \phi^{(1)i} D^a \phi_i^{(1)}) + D_{nm} (D_a \phi^{(1)i} D^a \phi_i^{(1)} TT^*) + h.c.] \right. \\ &\quad \left. - b_{n,m} [D'_{nm} (TD_a \phi^{(2)i} T^* D^a \phi_i^{(1)}) + D'_{nm} (D_a \phi^{(1)i} TD^a \phi_i^{(2)} T^*) + h.c.] \right), \\ L_2^{nm} &= \text{Tr} \left(a_{n,m} [D_{nm} (D^\alpha T D_\alpha T^* D_a \phi^{(1)i} D^a \phi_i^{(1)}) + D_{nm} (D_a \phi^{(1)i} D^a \phi_i^{(1)} D^\alpha T D_\alpha T^*) + h.c.] \right. \\ &\quad \left. - b_{n,m} [D'_{nm} (D^\alpha T D_a \phi^{(2)i} D_\alpha T^* D^a \phi_i^{(1)}) + D'_{nm} (D_a \phi^{(1)i} D_\alpha T D_a \phi_i^{(2)} D^\alpha T^*) + h.c.] \right), \\ L_3^{nm} &= -\text{Tr} \left(a_{n,m} [D_{nm} (D^\beta T D_\mu T^* D^\mu \phi^{(1)i} D_\beta \phi_i^{(1)}) + D_{nm} (D^\mu \phi^{(1)i} D_\beta \phi_i^{(1)} D^\beta T D_\mu T^*) + h.c.] \right. \\ &\quad \left. - b_{n,m} [D'_{nm} (D^\beta T D^\mu \phi^{(2)i} D_\mu T^* D_\beta \phi_i^{(1)}) + D'_{nm} (D^\mu \phi^{(1)i} D_\mu T D_\beta \phi_i^{(2)} D^\beta T^*) + h.c.] \right), \\ L_4^{nm} &= -\text{Tr} \left(a_{n,m} [D_{nm} (D^\beta T D^\mu T^* D_\beta \phi^{(1)i} D_\mu \phi_i^{(1)}) + D_{nm} (D^\beta \phi^{(1)i} D^\mu \phi_i^{(1)} D_\beta T D_\mu T^*) + h.c.] \right. \\ &\quad \left. - b_{n,m} [D'_{nm} (D^\beta T D_\beta \phi^{(2)i} D^\mu T^* D_\mu \phi_i^{(1)}) + D'_{nm} (D_\beta \phi^{(1)i} D_\mu T D^\mu \phi_i^{(2)} D^\beta T^*) + h.c.] \right) \end{aligned} \quad (64)$$

One might read off all the definitions of $D_{nm}(EFGH)$, $D'_{nm}(EFGH)$ from [43]. In order to actually produce all infinite singularities $(u + s' + t')$, one needs to employ the following rule

$$\mathcal{A} = V_\alpha^i(C_{p+1}, \phi) G_{\alpha\beta}^{ij}(\phi) V_\beta^j(\phi, \phi_1, T_2, T_2), \quad (65)$$

where the scalar propagator $G_{\alpha\beta}^{ij}(\phi) = \frac{-i\delta_{\alpha\beta}\delta^{ij}}{T_p(2\pi\alpha')^2(t'+s'+u)}$ can be derived from the scalar field's kinetic term that has been fixed in DBI action.

Let us introduce new coupling as

$$(2\pi\alpha')\mu_p \frac{1}{(p+1)!} \int_{\Sigma_{p+1}} \epsilon^{a_0 \dots a_p} C_{a_0 \dots a_{p-1}}^i D_{a_p} \phi^i \quad (66)$$

where in the above new coupling, the scalar field has been taken from pull back of brane, now let us derive its vertex operator in an EFT as

$$V_\alpha^i(C_{p+1}, \phi) = i(2\pi\alpha')\mu_p \frac{1}{(p+1)!} \epsilon^{a_0 \dots a_{p-1} b} p_b C_{a_0 \dots a_{p-1}}^i \text{Tr}(\lambda_\alpha). \quad (67)$$

where λ_α is an Abelian matrix. The off-shell's scalar field is Abelian and we need to consider two permutations as $\text{Tr}(\lambda_2 \lambda_3 \lambda_1 \lambda_\beta)$, $\text{Tr}(\lambda_2 \lambda_3 \lambda_\beta \lambda_1)$ to be able to derive $V_\beta^j(\phi, \phi_1, T_2, T_2)$ from (64) as below

$$\begin{aligned} V_\beta^j(\phi, \phi_1, T_2, T_2) &= \xi_1^j \frac{1}{2}(s')(t') (-2iT_p\pi)(\alpha')^{n+m+3} (a_{n,m} - b_{n,m}) \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_\beta) \\ &\quad \left((k_2 \cdot k)^n (k_1 \cdot k_2)^m + (k \cdot k_2)^n (k \cdot k_3)^m + (k_3 \cdot k)^n (k_2 \cdot k)^m + (k_3 \cdot k)^n (k_3 \cdot k_1)^m \right. \\ &\quad \left. + (k_1 \cdot k_3)^n (k_2 \cdot k_1)^m + (k_3 \cdot k_1)^n (k_3 \cdot k)^m + (k_2 \cdot k_1)^n (k \cdot k_2)^m + (k_2 \cdot k_1)^n (k_3 \cdot k_1)^m \right) \end{aligned} \quad (68)$$

where k is once more the momentum of off-shell scalar field and $b_{n,m}$'s are symmetric [24], lets list some of the coefficients as appeared ⁸ in [43]

$$\begin{aligned} a_{0,0} &= -\frac{\pi^2}{6}, b_{0,0} = -\frac{\pi^2}{12}, a_{1,0} = 2\zeta(3), a_{0,1} = 0, b_{0,1} = -\zeta(3), a_{1,1} = a_{0,2} = -7\pi^4/90, \\ a_{2,2} &= (-83\pi^6 - 7560\zeta(3)^2)/945, b_{2,2} = -(23\pi^6 - 15120\zeta(3)^2)/1890, a_{1,3} = -62\pi^6/945, \\ a_{2,0} &= -4\pi^4/90, b_{1,1} = -\pi^4/180, b_{0,2} = -\pi^4/45, a_{0,4} = -31\pi^6/945, a_{4,0} = -16\pi^6/945, \\ a_{1,2} &= a_{2,1} = 8\zeta(5) + 4\pi^2\zeta(3)/3, a_{0,3} = 0, a_{3,0} = 8\zeta(5), b_{1,3} = -(12\pi^6 - 7560\zeta(3)^2)/1890, \\ a_{3,1} &= (-52\pi^6 - 7560\zeta(3)^2)/945, b_{0,3} = -4\zeta(5), b_{1,2} = -8\zeta(5) + 2\pi^2\zeta(3)/3, \\ b_{0,4} &= -16\pi^6/1890. \end{aligned} \quad (69)$$

Now if we replace all the above vertices in the the above field theory sub amplitude we get to produce all order singularities in effective field theory as follows

$$2i\mu_p \frac{\epsilon^{a_0 \dots a_{p-1} b} \xi_i p_b C_{a_0 \dots a_{p-1}}^i}{(p+1)!(s'+t'+u)} \text{Tr}(\lambda_1 \lambda_2 \lambda_3) \sum_{n,m=0}^{\infty} (a_{n,m} - b_{n,m}) [s'^m t'^n + s'^n t'^m] s' t' \quad (70)$$

⁸ We have used $k_3 \cdot k = k_2 \cdot k_1 - k^2$, $k_2 \cdot k = k_1 \cdot k_3 - k^2$, also overlooked all the contact terms that k^2 caused.

Now let us compare all order EFT singularities with string theory poles that appeared in (61). First of all we try to cancel off all the common factors that appeared in both EFT and string theory side.

The next step is to set $n = m = 0$ so that from the string amplitude we get to find out the following coefficient $l_{0,0}s't' = -s't'\frac{\pi^2}{3}$ where as for the zeroth order of α' or leading singularity in the field theory amplitude we obtain

$$4s't'(a_{0,0} - b_{0,0}) = -s't'\frac{\pi^2}{3}$$

so as we can see both coefficient match at zeroth order . Let us keep counting . At first order of α' string amplitude proposes to us the the following coefficient $l_{1,0}s't'(s' + t') = 8\zeta(3)(s' + t')s't'$

while for the α' or next to the leading singularity in the field theory amplitude we gain the following term

$$2s't'(s' + t')(a_{1,0} + a_{0,1} - b_{0,1} - b_{0,1}) = 8\zeta(3)(s' + t')s't'$$

Let us compare at $(\alpha')^2$ order, so that the string amplitude imposes to us the following coefficient

$$l_{2,0}s't'(s' + t')^2 + l_{0,1}(s't')^2$$

meanwhile the field theory amplitude predicts the following counterparts

$$\begin{aligned} & 4(s't')^2(a_{1,1} - b_{1,1}) + 2s't'(a_{0,2} + a_{2,0} - b_{0,2} - b_{2,0})[(s')^2 + (t')^2] \\ &= s't'(-\frac{7\pi^4}{45}(s' + t')^2 + \frac{\pi^4}{45}s't') = (l_{2,0}(s' + t')^2 + l_{0,1}s't')s't' \end{aligned}$$

One can check to see the other orders are also matched and this clearly confirms that we have been able to precisely produce all infinite $(u + s' + t')$ bulk singularities of string amplitude in an effective field theory side where the important points were to look for new coupling that has been shown up in (66) and also the fact that all order α' higher derivative corrections of two scalars and two tachyons of brane anti brane are exact and correct and do differ from the corrections of non-BPS branes. Notice that all terms like $D\phi^{(1)i}.D\phi_i^{(2)}$'s have to appear inside the all order α' corrections, for instance , look at the $b_{n,m}$ coefficients

in (64). Ultimately all order contact interactions could be explored from the last section of [27].

Note that the nature of the $(s' + t' + u)$ poles can be understood as follows. It is a closed string that is absorbed by the brane and becomes an excited open string that later on decays into infinite n massless scalar strings. The case $n = 0$ is pure absorption, $n = 1$ is mixing and $n = 2$ is the origin of Hawking radiation and we have studied all the other cases. Similar poles appear also in scattering of closed strings off D-branes [46]. One might be also interested in exploring several consistency conditions for branes, various anomalies, tadpoles, and topological couplings where we recommend some relevant papers in [47] and references therein.

Now we do want to deal with all the poles in \mathcal{A}_2 , note that in this case $\epsilon^{a_0 \dots a_p} p^i C_{a_0 \dots a_p} = \epsilon^{a_0 \dots a_p} H_{a_0 \dots a_p}^i$ and in order to produce the first bulk singularities, one needs to consider the following Feynman rule as

$$\begin{aligned} \mathcal{A} = & V_i(C_{p+1}, \phi^{(1)}) G_{ij}(\phi) V_j(\phi^{(1)}, T_1, T_1, \phi^{(1)}) \\ & + V_i(C_{p+1}, \phi^{(2)}) G_{ij}(\phi) V_j(\phi^{(2)}, T_1, T_1, \phi^{(1)}) \end{aligned} \quad (71)$$

also the Taylor expansion of scalar field through Chern- Simons coupling is needed as below

$$\frac{1}{(p+1)!} (2\pi\alpha') \int_{\Sigma_{p+1}} \partial_i C_{a_0 \dots a_p} (\phi_1^i - \phi_2^i) \epsilon^{a_0 \dots a_p}$$

where ϕ in the propagator can be $\phi^{(1)}$ and $\phi^{(2)}$ and all the effective field theory vertices are

$$\begin{aligned} G_{ij}(\phi) &= \frac{i\delta_{ij}\delta_{\alpha\beta}}{(2\pi\alpha')^2 T_p (u + t' + s')} \\ V_i(C_{p+1}, \phi^{(1)}) &= i\mu_p (2\pi\alpha') \frac{1}{(p+1)!} \epsilon^{a_0 \dots a_p} p^i C_{a_0 \dots a_p} \text{Tr}(\lambda_\alpha) \\ V_i(C_{p+1}, \phi^{(2)}) &= -i\mu_p (2\pi\alpha') \frac{1}{(p+1)!} \epsilon^{a_0 \dots a_p} p^i C_{a_0 \dots a_p} \text{Tr}(\lambda_\alpha) \\ V_j(\phi^{(1)}, T_1, T_1, \phi^{(1)}) &= -2iT_p (2\pi\alpha') \xi_j \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_\beta) \\ V_j(\phi^{(2)}, T_1, T_1, \phi^{(1)}) &= 2iT_p (2\pi\alpha') \text{Tr}(\lambda_1 \lambda_2 \lambda_3 \lambda_\beta) \xi_j \end{aligned} \quad (72)$$

If we apply all the vertices into (71) which is the effective field theory rule we obtain

$$\mathcal{A} = \frac{4i\mu_p}{(p+1)!(u+s'+t')} \epsilon^{a_0 \dots a_p} p^i C_{a_0 \dots a_p} \xi^i \quad (73)$$

thus we have produced the first $(u+s'+t')$ singularity pole of \mathcal{A}_2 . Note that, this section does clarify the fact that the following interaction coupling term

$$D\phi^{(1)i}.D\phi_i^{(2)} \quad (74)$$

has to be appeared in an EFT. Indeed if we do not take into account $D\phi^{(1)i}.D\phi_i^{(2)}$ term in an effective field theory then $V_j(\phi^{(2)}, T_1, T_1, \phi^{(1)})$ will not have any contribution to field theory and hence we are no longer able to even start producing the first simple scalar pole of string amplitude in the effective field theory.

The important point that is worth highlighting is that the new coupling (74) is not embedded inside the ordinary trace effective action. Eventually in sections 3.2 and 3.3 of [27] we have shown how to get to all infinite scalar poles as well as all the contact interactions.

Finally it would be remarkable to find out the S-matrix of one closed string RR and four tachyons in the world volume of brane anti brane system, where by doing so, one might be able to make various remarks on the part of the symmetrized trace tachyonic DBI effective action [48] as well as explore some kinds of different singularity structures. We hope to come over those open questions in near future.

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